APPENDIX I

A Simple Model of Recurrent Cost Disequilibrium

Let

V_t = Quantity of variable input at time t into public production (e.g., labor, materials, etc.)

K_t = Quantity of fixed Input at time t into public production (e.g., building, roads, etc.)

Q_t = Quantity of government services produced at time t (e.g., # of student educated).

R_t = Total government revenue available at time t

I_t = The amount of revenue allocated to new investment in fixed inputs at time t.

W_t = price of variable input

P_t = price of the fixed input

At any time, e.g., t =1, the government must decide how much of its revenue should be allocated to purchasing variable inputs, and how much should be allocated to purchasing new capital. Let us assume the following:

(1) The price of government services is the numeraire good, equal to one. Thus Q, the quantity of government output is also the value of government output.

(2) Output is determined by the following generalized production function; which we will assume is differentiable.

(1) $Q_t = f(K_t, V_t)$

where

(2) $V_t = \frac{R_t - P_t I_t}{W_t}$

Now assume that the objective function of the government is to maximize the present value of the stream of government output subject to the production function and total revenue constraints.

Let Q = the discount rate of the government

Then

(3) $Q = \sum_{i=1}^{T} \frac{f(K_t, V_t)}{(1+ Q)^t}$ where $K_t = K_{t-1} = \frac{I_t}{P_t}$ and $V_t = \frac{R_t - P_t I_t}{W_t}$

or

(4) $Q = \sum_{t=1}^{T} \frac{f(K_{t-1} + I_t \quad R_t - P_t I_t}{(1 = Q)^t}$

or

$$(5)\qquad \overset{T}{\underset{t=1}{£}} \quad f(\overset{t-1}{\underset{1=0}{£}} K_0 + \frac{I_i}{P_i} \;,\; \frac{R_t - P_t}{W_t} \;/\;) \;/(1+Q)^t$$

Taking the partial derivative of Q with respect to I_j, and setting the result equal to zero, we get

$$(6)\qquad \frac{-P_j f_{2j}}{W_j(1+Q)^j} + \overset{T}{\underset{t=j+1}{£}} \frac{f_{1t}}{P_j(1+Q)^t} = 0$$

$$(7)\qquad \frac{P_j}{W_j} \quad \frac{f_{2j}}{(1+Q)^j} = \overset{T}{\underset{t=j+1}{£}} \frac{f_{1t}}{P_j(1+Q)^t}$$

where

$$f_{1t} = \frac{6Q_t}{6K_t} \quad ; \quad f_{2j} = \frac{6Q_j}{6V_j}$$

The interpretation of the result seems straightforward. The government should allocate resources until the present value of the marginal value product of an additional unit of the variable input is exactly equal to the present value of an additional unit of fixed inputs.

When this decision rule is not followed, either because of LDC government policy which underallocates resources to the variable inputs, or because donors limit the fungibility of their assistance making it impossible for LDC's to allocate resources efficiently, then there is a recurrent cost problem.

Note that there are really two allocation decisions. The first concerns the amount of resources which is to be allocated to public production as opposed to private production. Up to this point we have not investigated that question in this Appendix. The second allocation decision is the choice between fixed and variable inputs, and in terms of our model, a recurrent cost problem exists when

$$(8)\qquad \frac{P_j f_{2j}}{W_j(1+Q)^j} > \overset{T}{\underset{t=j+1}{£}} \frac{f_{it}}{P_j(1+Q)^t}$$

Let us now consider the allocation of resources between public and private production. It is clear that by allowing revenues to vary, so that the government can compete for scarce resources, the optimal allocation rule is:

$$(9)\quad PVMPV^g_t = PVMPV^p_t; \quad PVMPK^g_t = PVMPK^p_t$$

Where

$PVMPV^g_t$= the present value of the marginal product of an extra unit of variable input into government production

$PVMPK^g_t$= the present value of the marginal product of an extra unit of fixed input into government production

$PVMPV^p_t$, $PVMPK^p_t$ have similar meanings for private production thus the present value of the marginal product of any input should be equal in the public and private sectors.

If government cannot claim enough resources so that its marginal production is as valuable as that of the private sector, then resource allocation is not optimal. The same result holds if the government claims too many resources.

From the point of view of project analysis we are interested in both allocation questions. If either

$$PVMPV^g_t > PVMPV^g_t \tag{10}$$

$$\frac{PVMPV^g_t}{W_t} > \frac{PVMPK^g_t}{P_t}$$

then the shadow price of government revenue is greater than one.

In calculating rates of return for new projects, the use of such a shadow price recurrent expenditures and revenues will enable one to make investment decisions so as to economize on recurrent resources (cf. Appendix II for a detailed example).

The model is easily expanded to include three inputs: a fixed input, a locally produced variable input such as labor, and an imported variable input such as petrol.